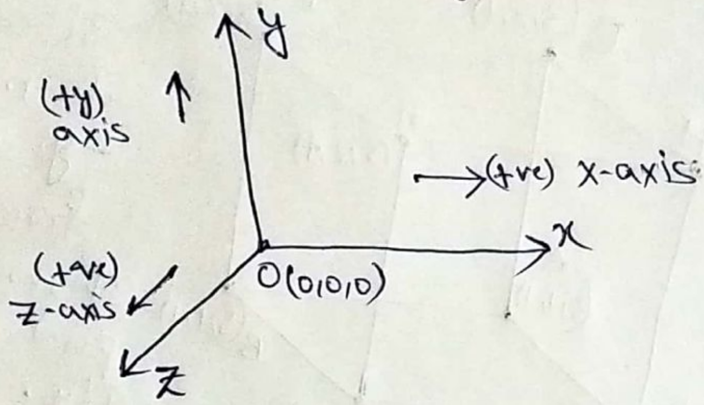
1. **Three-dimensional Geometry**

Class -12th

By: Mr. Niraj Keshri
B.Tech. IIT Kharagpur

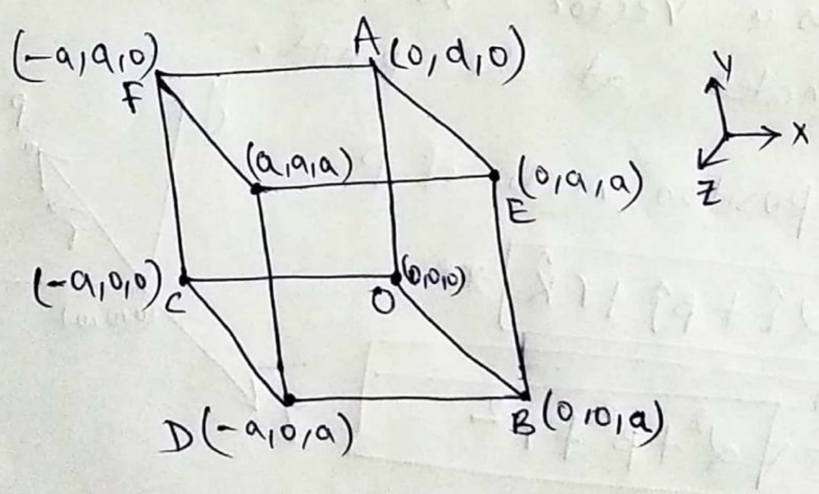
1. Fundamentals:-

1.1 Rectangular Co-ordinate System:-



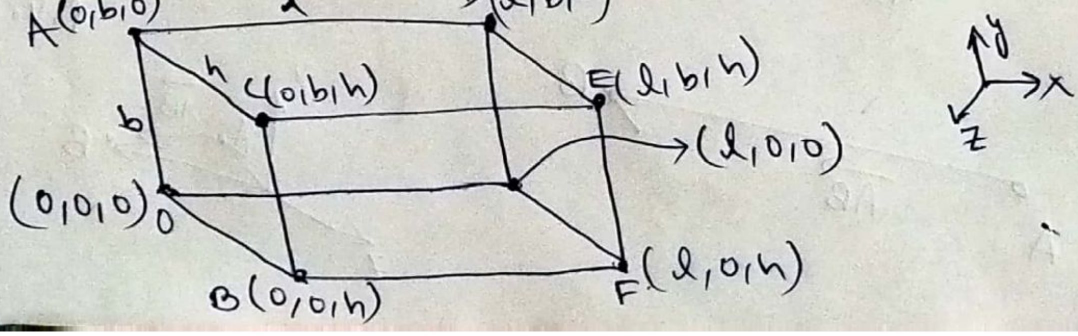
Example:- 1: Find all the vertices of a cube whose side length is 'a' and take any one vertices as origin.

Solution:-



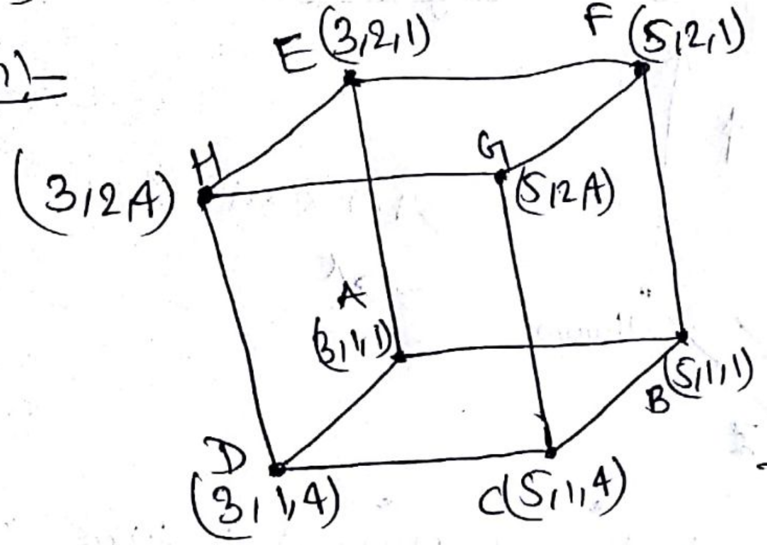
Example:- 2 Find the vertices of a cuboid whose dimension is given as $(l \times b \times h)$ in three-dimensional space taking any one of the vertices as origin.

Solution:-



Example :- 3 Find the remaining vertices of the Cuboid which is constructed from (3,2,4) and by travelling in $(+x, -y, -z)$ direction by $(2, 1, 3)$ units:

Solution:-

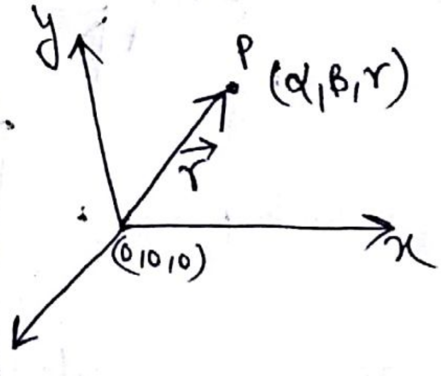


Since we have to move in (+ve)-x. -y. and (-ve)-z axis from point (3,2,4) so, we choose the point (3,2,4) accordingly.

1.2 position vector of a point in a space.

It is a vector which starts from origin and terminates at the point.

$\vec{r}$ = position vector of P



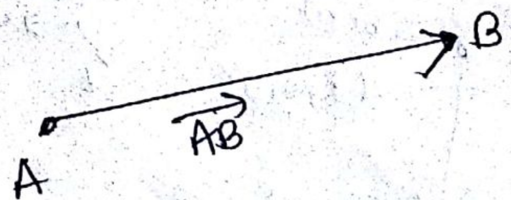
$$\boxed{\vec{r} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}}$$ **

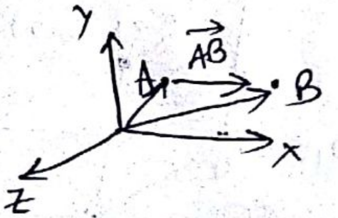
$$\boxed{|\vec{r}| = \sqrt{\alpha^2 + \beta^2 + \gamma^2}}$$

If we have two position vector A and B. then $\vec{AB}$ will be given as :-

$\vec{AB}$ = (P.v. of B) − ~~(P.V.)~~ (P.v. of A)

P.v. = position vector.





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Example:- 4

find the position vector for a point $P(2,3,4)$ in the Space:

Solution:- $\vec{r} = \vec{OP} = 2\hat{i}+3\hat{j}+4\hat{k}$.

Example:- 5

find the vector $\vec{PQ}$ If coordinates of the points are $P(2,5,7)$ and $Q(3,9,11)$.

Solution:

$$\vec{PQ} = (\text{p.v. of } Q) - (\text{p.v. of } P)$$

$$= (3\hat{i}+9\hat{j}+11\hat{k}) - (2\hat{i}+5\hat{j}+7\hat{k})$$

$$\vec{PQ} = \hat{i}+4\hat{j}+4\hat{k} \quad \text{Ans}$$

1.3. Distance between two point in a space:-

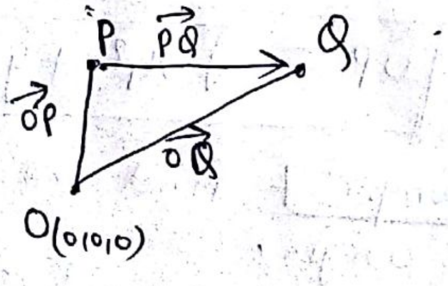
Suppose we have two points in a space given as $P(x_1,y_1,z_1)$ and $Q(x_2,y_2,z_2)$.

$$\therefore \vec{PQ} = (\text{p.v. of } Q) - (\text{p.v. of } P)$$

$$= (x_2\hat{i}+y_2\hat{j}+z_2\hat{k}) - (x_1\hat{i}+y_1\hat{j}+z_1\hat{k})$$

$$\vec{PQ} = (x_2-x_1)\hat{i}+(y_2-y_1)\hat{j}+(z_2-z_1)\hat{k}$$

Now, we have to find the distance between P and Q. which can be given by $|\vec{PQ}|$



$$\therefore |\vec{PQ}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

Hence, $PQ = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

PQ = distance b/w P and Q.

Example:- 6 find the distance between two points given as $P(2,1,1)$ and $Q(-1,3,-1)$.

Solution:-

~~using~~ Using the distance formula, we have

$x_1 = 2$ and $x_2 = -1$

$y_1 = 1$ $\quad$ $y_2 = 3$

$z_1 = 1$ $\quad$ $z_2 = -1$

we get $PQ = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

$$= \sqrt{(-1-2)^2 + (3-1)^2 + (-1-1)^2}$$

$$= \sqrt{(-3)^2 + (2)^2 + (-2)^2}$$

$$= \sqrt{9+4+4}$$

$$\boxed{PQ = \sqrt{17}}$$ Ans

1.4 Distance of a point from origin:-

Using distance formula if we will put $Q(x_2, y_2, z_2)$ as origin i.e. $x_2 = 0$, $y_2 = 0$. and $z_2 = 0$, we will get distance of $P(x_1, y_1, z_1)$ from origin as

$$OP = \sqrt{(0-x_1)^2 + (0-y_1)^2 + (0-z_1)^2}$$

$$\boxed{OP = \sqrt{x_1^2 + y_1^2 + z_1^2}}$$

Example:-7 find the distance of the point $P(1,1,1)$ from origin.

Solution:- $OP = \sqrt{1^2+1^2+1^2}$

$$\boxed{OP = \sqrt{3}}$$ Ans

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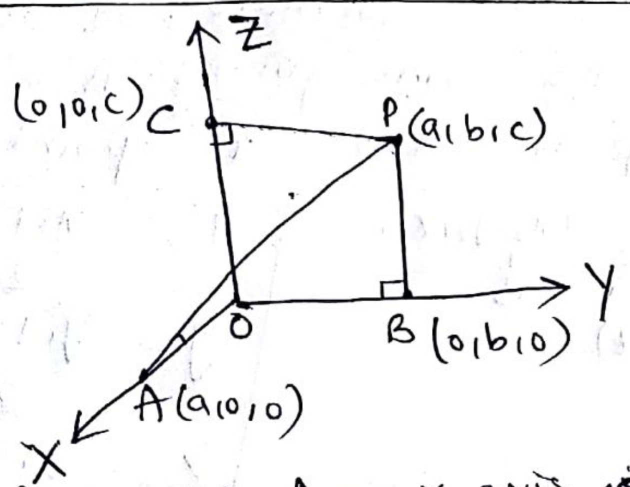
1.5. Distance of a point from coordinate axes:-

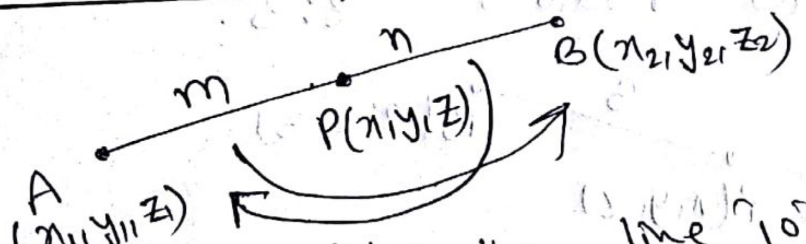


The distance of a point from x-axis = $\sqrt{b^2+c^2}$

The distance of a point from y-axis = $\sqrt{a^2+c^2}$

The distance of a point from z-axis = $\sqrt{a^2+b^2}$

2.1. Section formula in 3D geometry:-



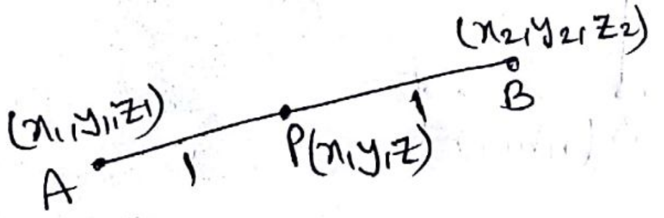
If a point p divides the line joining A and B in the ratio of m:n, then the coordinate of p will be given by

$$\boxed{x = \frac{mx_2+nx_1}{m+n} \qquad y = \frac{my_2+ny_1}{m+n} \qquad z = \frac{mz_2+nz_1}{m+n}}$$

The only difference b/w 2D and 3D section formula is, we have got one more axis (Z)...

2.2 Mid-point formula in 3-D Geometry:-

If in the Section formula, if the point P divides the the line joining A and B in 1:1 ratio the P is the midpoint of A and B.



$$x = \frac{x_1 + x_2}{2}$$

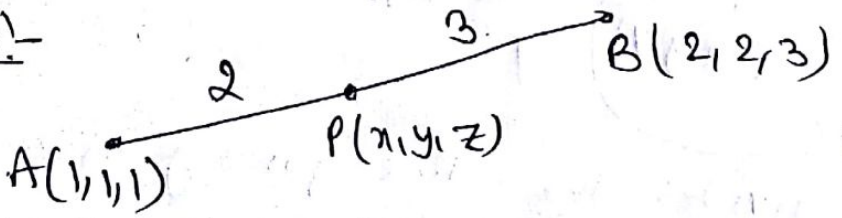
$$y = \frac{y_1 + y_2}{2}$$

$$z = \frac{z_1 + z_2}{2}$$

We got the formula just by putting $\boxed{m=1}$ and $\boxed{n=1}$ in section formula.

Example:- 8 Find the coordinates of the point P which divides the line joining A(1, 1, 1) and B(2, 2, 3) in the ratio of 2:3.

Solution:-



$\therefore$ Using section formula:-

$$x = \frac{2 \times 2 + 3 \times 1}{2+3} = \frac{4+3}{5} = 7/5$$

$$y = \frac{2 \times 2 + 3 \times 1}{2+3} = \frac{4+3}{5} = 7/5$$

$$z = \frac{2 \times 3 + 3 \times 1}{2+3} = \frac{6+3}{5} = 9/5$$

$\therefore$ the coordinate of P is $P\left(\frac{7}{5}, \frac{7}{5}, \frac{9}{5}\right)$. Ans

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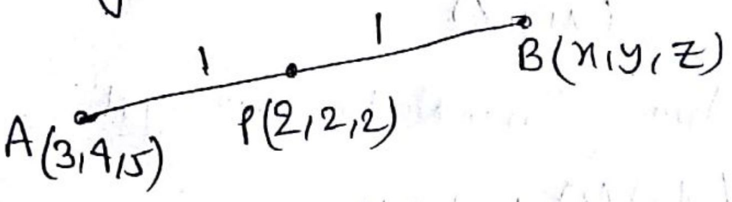
Example:- 9 If the point $P(2,2,2)$ is the midpoint of $A(3,4,5)$ and B. What is the coordinate of B = ?

Solution:- Using midpoint formula:-



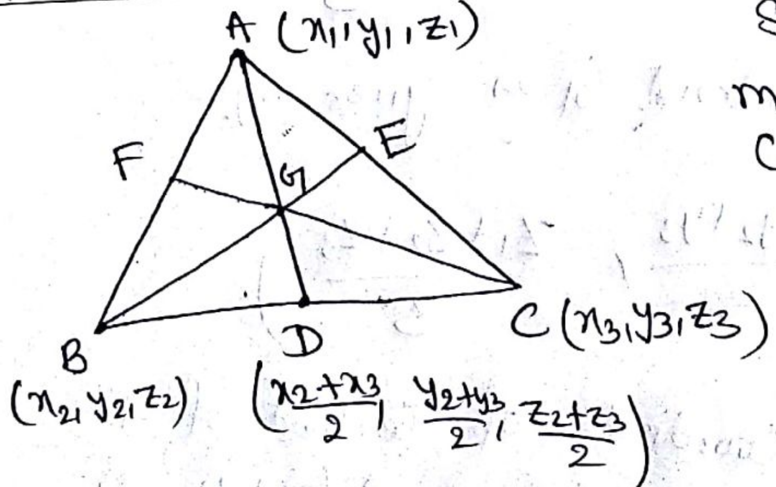
$$2 = \frac{3+x}{2} \Rightarrow 4 = 3+x \Rightarrow x = 4-3 = 1$$

$$2 = \frac{4+y}{2} \Rightarrow 4 = 4+y \Rightarrow y = 4-4 = 0$$

$$2 = \frac{5+z}{2} \Rightarrow 4 = 5+z \Rightarrow z = 4-5 = -1$$

$\therefore$ Coordinate of point B is $B(1,0,-1)$. Ans

2.3. Centroid of a triangle:-



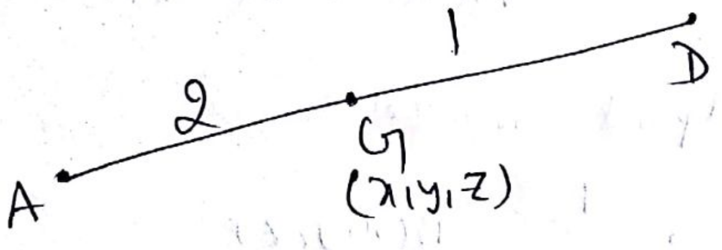
Since AD, BE and CF are median and G is the centroid of ΔABC, Hence D, E and F are the midpoint of BC, AC and AB respectively.

Hence, we will get the ~~Coordi~~ coordinate of D using midpoint formula.

Now, we know that centroid divides the median in the ratio of 2:1. Thus knowing coordinate of A and D, we can calculate the coordinate of G (centroid) using section formula.

$\therefore$ Coordinates of $D \equiv \left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}, \frac{z_2+z_3}{2} \right)$

and $A \equiv (x_1, y_1, z_1)$.



Applying section formula:- we got:

$$x = \frac{2 \times \left(\frac{x_2+x_3}{2} \right) + 1 \times x_1}{2+1} = \frac{x_1+x_2+x_3}{3}$$

$$y = \frac{2 \times \left(\frac{y_2+y_3}{2} \right) + 1 \times y_1}{2+1} = \frac{y_1+y_2+y_3}{3}$$

$$z = \frac{2 \times \left(\frac{z_2+z_3}{2} \right) + 1 \times z_1}{2+1} = \frac{z_1+z_2+z_3}{3}$$

$\therefore$ Coordinates of Centroid G is given by:-

$$G \equiv \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3} \right).$$

__Example:- 9__ Find the coordinate of centroid of the triangle whose vertices are given by $A(0,1,1)$, $B(1,2,3)$, $C(-1,-3,-5)$.

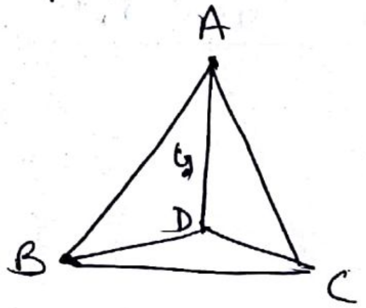
__Solution:-__ $G \equiv \left(\frac{0+1-1}{3}, \frac{1+2-3}{3}, \frac{1+3-5}{3} \right)$

$G \equiv \left(0, 0, -\frac{1}{3} \right)$ Ans

2.4. Centroid of Tetrahedron:-

Let the co-ordinates of A, B, C and D are (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3) and (x_4, y_4, z_4) respectively. Then the coordinates of its, ~~vertices are~~:- Centroid is:-

$$G \equiv \left(\frac{x_1+x_2+x_3+x_4}{4}, \frac{y_1+y_2+y_3+y_4}{4}, \frac{z_1+z_2+z_3+z_4}{4} \right).$$



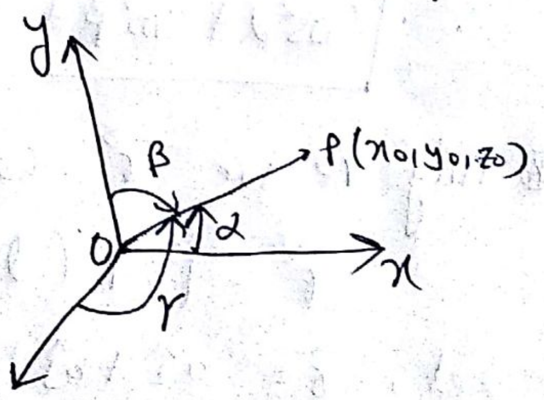
3.1. Direction cosines of a vector or a line:-

If a line makes an angle α, β, γ with the positive direction of the coordinates axes. Then $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are known as the direction cosines of the given line and are generally denoted as l, m and n respectively.

Thus, $l = \cos\alpha$, $m = \cos\beta$ and $n = \cos\gamma$.

$$\therefore \overrightarrow{OP} = x_0\hat{i} + y_0\hat{j} + z_0\hat{k}$$

$$|\overrightarrow{OP}| = \sqrt{x_0^2 + y_0^2 + z_0^2}.$$



Now

$$\cos\alpha = \frac{\overrightarrow{OP} \cdot \hat{i}}{|\overrightarrow{OP}| \cdot |\hat{i}|} \quad \text{[using dot product]}$$

$$\boxed{\cos\alpha = \frac{x_0}{\sqrt{x_0^2 + y_0^2 + z_0^2}} = l}$$

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Similarly,

$$\cos\beta = \frac{\vec{OP}\cdot\hat{j}}{|\vec{OP}|\cdot|\hat{j}|} = \frac{y_0}{\sqrt{x_0^2+y_0^2+z_0^2}} = m$$

$$\cos\gamma = \frac{\vec{OP}\cdot\hat{k}}{|\vec{OP}|\cdot|\hat{k}|} = \frac{z_0}{\sqrt{x_0^2+y_0^2+z_0^2}} = n$$

3.2. Results on Direction Cosines:-

$$\cos^2\alpha = l^2 = \frac{x_0^2}{x_0^2+y_0^2+z_0^2}$$

$$\cos^2\beta = m^2 = \frac{y_0^2}{x_0^2+y_0^2+z_0^2}$$

$$\cos^2\gamma = n^2 = \frac{z_0^2}{x_0^2+y_0^2+z_0^2}$$

Adding, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = l^2+m^2+n^2$

$$= \frac{x_0^2+y_0^2+z_0^2}{x_0^2+y_0^2+z_0^2} = 1$$

$$\therefore \boxed{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = l^2+m^2+n^2 = 1}$$

* Any vector $\vec{r}$ can be represented as:-

$$\vec{r} = |\vec{r}|\,(l\hat{i}+m\hat{j}+n\hat{k})$$

Let, $\vec{r} = a\hat{i}+b\hat{j}+c\hat{k}$

$$\therefore \frac{\vec{r}}{|\vec{r}|} = \frac{a}{|\vec{r}|}\hat{i} + \frac{b}{|\vec{r}|}\hat{j} + \frac{c}{|\vec{r}|}\hat{k}$$

$$\vec{r} = |\vec{r}|\,(l\hat{i}+m\hat{j}+n\hat{k})$$

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* If a vector $\vec{r}$ having direction cosines l, m, n the projection of $\vec{r}$ on the coordinates axes are given by:- $l|\vec{r}|, m|\vec{r}|, n|\vec{r}|$.

* The projection of the segment joining the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ on a line having direction cosines l, m, n is given by:-
$l(x_2-x_1) + m(y_2-y_1) + n(z_2-z_1)$

* Direction cosines of the Axes:-
Since the positive x-axis makes $0°, 90°, 90°$ with the (+ve) axes of x, y and z respectively, then the direction cosines of x-axis are $(1, 0, 0)$
Similarly, the direction cosines of y-axis and z-axis are $(0, 1, 0)$ and $(0, 0, 1)$ respectively.

3.3. Direction Ratios of a vector or a line:-

A set of three numbels a, b, c which are proportional to the direction cosines l, m, n respectively of a line, are called the direction ratios. Thus, $\frac{l}{a} = \frac{m}{b} = \frac{n}{c}$

or, $a = \lambda l$
$b = \lambda m$
$c = \lambda n$

Example:- 10 Find the direction Cosine of a point $p(2,3,4)$.

Solution:- $\vec{OP} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$$|\vec{OP}| = \sqrt{2^2 + 3^2 + 4^2}$$
$$= \sqrt{4+9+16} = \sqrt{29}$$

$$\therefore DC's = \frac{2}{\sqrt{29}}, \frac{3}{\sqrt{29}}, \frac{4}{\sqrt{29}}$$
$$l, \quad m, \quad n$$

Example:- 11 Find the Direction Cosine and Direction Ratios of a vector $2\hat{i} - 2\hat{j} + \hat{k}$.

Solution:-

$$\vec{r} = 2\hat{i} - 2\hat{j} + \hat{k}$$
$$|\vec{r}| = \sqrt{(2)^2 + (-2)^2 + (1)^2}$$
$$= \sqrt{4+4+1} = 3$$

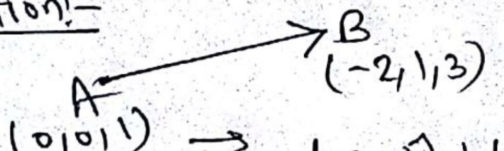
$$\therefore DC's \equiv \left(\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}\right)$$

$$DR's \equiv \left(\frac{2}{3}\lambda, -\frac{2}{3}\lambda, \frac{1}{3}\lambda\right)$$

Example:- 12 Find the direction cosine of the line joining A to B. $A \equiv (0,0,1)$

$B \equiv (-2,1,3)$

Solution:-



$$\therefore DC's \equiv \left(-\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$$

$$\vec{AB} = (-2-0)\hat{i} + (1-0)\hat{j} + (3-1)\hat{k}$$
$$\vec{AB} = -2\hat{i} + \hat{j} + 2\hat{k}$$
$$|\vec{AB}| = \sqrt{(-2)^2 + 1^2 + 2^2} = 3$$

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Example:- 13 If α, β, γ are the angles made by the line with the positive coordinate axes. Find the value of:- $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = ?$

Solution:- As we ~~know~~ know that:-

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ [$\cos\alpha, \cos\beta, \cos\gamma$ are Direction Cosines]

$\therefore (1-\sin^2\alpha) + (1-\sin^2\beta) + (1-\sin^2\gamma) = 1$ [$\because \sin^2\theta + \cos^2\theta = 1$]

$\therefore 3 - (\sin^2\alpha + \sin^2\beta + \sin^2\gamma) = 1$

$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 3 - 1 = 2$ Ans

Example:- 14 A line makes the same angle θ, with each of the x and z axis. If the angle β, which ~~m~~ it makes with y-axis, is such that $\sin^2\beta = 3\sin^2\theta$, then $\cos^2\theta$ equals:-

Solution:- As we know that:-

$l^2 + m^2 + n^2 = 1$ where, l, m, n are DC's.

$\therefore \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ [$\because \alpha = \gamma = \theta$]

$\Rightarrow \cos^2\theta + \cos^2\beta + \cos^2\theta = 1$

$\Rightarrow 2\cos^2\theta = 1 - \cos^2\beta$

$\Rightarrow 2\cos^2\theta = \sin^2\beta$

$\Rightarrow 2\cos^2\theta = 3\sin^2\theta$ [$\because \sin^2\beta = 3\sin^2\theta$]

$\Rightarrow 2\cos^2\theta = 3(1-\cos^2\theta)$

$\Rightarrow 2\cos^2\theta + 3\cos^2\theta = 3$ $\Rightarrow$ $\boxed{\cos^2\theta = \frac{3}{5}}$ Ans

Example:- 15 If $\cos\alpha, \cos\beta, \cos\gamma$ are the Direction cosines (DC's) of a line which is equally inclined to the coordinates axes, then find the value of $\tan^2\alpha + \tan^2\beta + \tan^2\gamma = ?$

Solution:- Since $\alpha = \beta = \gamma$ [given]

$\therefore \cos\alpha = \cos\beta = \cos\gamma$

$\therefore \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow 3\cos^2\alpha = 1$

$\cos^2\alpha = 1/3, \quad \sin^2\alpha = 1 - 1/3 = 2/3$

$\therefore \tan^2\alpha + \tan^2\beta + \tan^2\gamma = 3\tan^2\alpha$

$\Rightarrow 3 \cdot \dfrac{\sin^2\alpha}{\cos^2\alpha} = 3 \times \dfrac{2 \times \cancel{3}}{\cancel{3} \times 1}$

$= 6$. Ans

3.4. Angle between two vectors in terms of DC's:-

Let $\vec{m} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$

$\vec{n} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$

then $\vec{m}\cdot\vec{n} = |\vec{m}||\vec{n}|\cos\theta$ $\quad \theta =$ Angle b/w $\vec{m}$ & $\vec{n}$.

$$\therefore \cos\theta = \frac{\vec{m}\cdot\vec{n}}{|\vec{m}||\vec{n}|} = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\lambda_1 \cdot \lambda_2}$$, where:- $\lambda_1 = \sqrt{a_1^2+b_1^2+c_1^2}$

$\lambda_2 = \sqrt{a_2^2+b_2^2+c_2^2}$

$$= \frac{a_1}{\lambda_1}\cdot\frac{a_2}{\lambda_2} + \frac{b_1}{\lambda_1}\cdot\frac{b_2}{\lambda_2} + \frac{c_1}{\lambda_1}\cdot\frac{c_2}{\lambda_2}$$

$\boxed{\cos\theta = l_1l_2 + m_1m_2 + n_1\cdot n_2}$ where, l_1, m_1, n_1 and l_2, m_2, n_2 are DC's of $\vec{m}$ and $\vec{n}$ respectively.

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* Condition of perpendicularity:-

Here, $\theta = 90^\circ$, $\therefore \cos 90^\circ = 0$.

Hence, $\boxed{l_1 l_2 + m_1 m_2 + n_1 n_2 = 0}$

* Condition of parallelism:-

Here, $\theta = 0$, $\therefore \cos 0^\circ = 1$

$\therefore l_1 l_2 + m_1 m_2 + n_1 n_2 = 1$

$(l_1 l_2 + m_1 m_2 + n_1 n_2)^2 = 1$

$\Rightarrow \cancel{(l_1 l_2 + m_1 m_2 + n_1 n_2)^2} = (l_1^2 + m_1^2 + n_1^2)(l_2^2 + m_2^2 + n_2^2) - 1 = 0$

$\Rightarrow (l_1 m_2 - l_2 m_1)^2 + (m_1 n_2 - n_2 m_1)^2 + (n_1 l_2 - l_2 n_1)^2 = 0$

$\Rightarrow (l_1 m_2 - l_2 m_1) = 0$; $(m_1 n_2 - n_2 m_1) = 0$; $(n_1 l_2 - l_2 n_1) = 0$

$\Rightarrow \boxed{\dfrac{l_1}{l_2} = \dfrac{m_1}{m_2} = \dfrac{n_1}{n_2}}$

Example-16 If two lines having direction cosines as : $(1, 2, 3)$ and $(3, 5, \alpha)$ are perpendicular then find $\alpha = ?$

Solution:- For $\perp$r/perpendicular lines.

$$l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$$

$$\Rightarrow 1 \cdot 3 + 2 \cdot 5 + 3 \cdot \alpha = 0$$

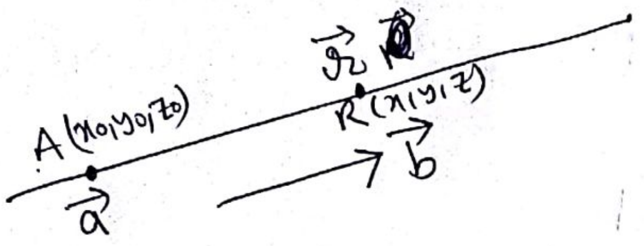
$$\Rightarrow 3 + 10 + 3\alpha = 0$$

$$3\alpha = -13$$

$$\alpha = -\frac{13}{3} \text{ Ans}$$

Straight line:-

4.1. Equation of a straight line passing through a point and parallel to a vector (in vector form as well as Cartesian/symmetric form):-



Given a line containing a specific point A(x_0, y_0, z_0) having position vector $\vec{a}$, and a vector $\vec{b}$ parallel to the line.

O (origin)

We have taken a general point R(x, y, z) having position vector $\vec{r}$.

$\therefore$ $\overrightarrow{AR}$ is parallel to $\vec{b}$.

$\therefore$ $\overrightarrow{AR} \parallel \vec{b}$ λ is an arbitary constant

$\therefore$ $\boxed{\overrightarrow{AR} = \lambda\vec{b}}$

$\therefore$ $\vec{r} - \vec{a} = \lambda\vec{b}$

$\boxed{\vec{r} = \vec{a} + \lambda\vec{b}}$ Equation of line in vector form.

Now, since $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{a} = x_0\hat{i} + y_0\hat{j} + z_0\hat{k}$

$\vec{b} = a\hat{i} + b\hat{j} + c\hat{k}$.

$\therefore$ $x\hat{i} + y\hat{j} + z\hat{k} = x_0\hat{i} + y_0\hat{j} + z_0\hat{k} + \lambda(a\hat{i} + b\hat{j} + c\hat{k})$

$\Rightarrow$ $x\hat{i} + y\hat{j} + z\hat{k} = (x_0 + \lambda a)\hat{i} + (y_0 + \lambda b)\hat{j} + (z_0 + \lambda c)\hat{k}$

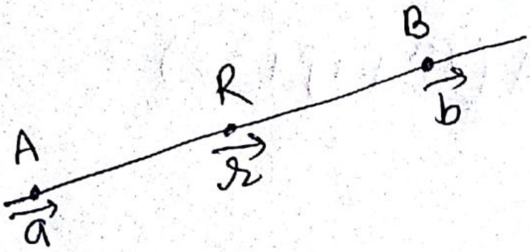
$\Rightarrow$ $x = x_0 + a\lambda$, $y = y_0 + b\lambda$, $z = z_0 + c\lambda$

$\Rightarrow$ $$\boxed{\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c} = \lambda}$$

Equation of line in symmetric form:

By:- Niraj Keshri
B. Tech. IIT Kharagpur

4.2. Equation of Straight line passing through two points :-



Let say, we have two points $A \equiv (x_1, y_1, z_1)$ and $B \equiv (x_2, y_2, z_2)$ given on the line.
We assumed a general point $R \equiv (x, y, z)$ on the line,

Now, we can see that
$\vec{AR}$ is parallel to $\vec{AB}$.

$$\therefore \vec{AR} = \lambda \vec{AB}$$

$$\vec{r} - \vec{a} = \lambda (\vec{b} - \vec{a})$$

$$\boxed{\vec{r} = \vec{a} + \lambda (\vec{b} - \vec{a})}$$ vector equation of a line passing through two points.

If we take, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$

$$\vec{b} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$$

$$\Rightarrow x\hat{i} + y\hat{j} + z\hat{k} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k} + \lambda\left((x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}\right)$$

$$\Rightarrow x\hat{i} + y\hat{j} + z\hat{k} - (x_1\hat{i} + y_1\hat{j} + z_1\hat{k}) = \lambda\left((x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}\right)$$

$$\Rightarrow (x - x_1)\hat{i} + (y - y_1)\hat{j} + (z - z_1)\hat{k} = \lambda\left[(x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}\right]$$

$$\therefore \boxed{\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1} = \lambda}$$

Symmetric equation of line.

Example: 17 Convert the symmetric equation of line in vector form:

i) $\dfrac{x-1}{2} = \dfrac{y-4}{1} = \dfrac{z-3}{3}$

Solution:- $\vec{r} = 1\hat{i} + 4\hat{j} + 3\hat{k} + \lambda(2\hat{i} + 1\hat{j} + 3\hat{k})$

ii) $\dfrac{x+1}{5} = \dfrac{y+3}{4} = \dfrac{z}{-5}$

Solution:- $\vec{r} = -1\hat{i} - 3\hat{j} + 0\hat{k} + \lambda(5\hat{i} + 4\hat{j} - 5\hat{k})$

iii) $\dfrac{x-1}{2} = \dfrac{2y-1}{2} = \dfrac{z-2}{3}$

Solution:- $\dfrac{x-1}{2} = \dfrac{y-1/2}{1} = \dfrac{z-2}{3}$

$\vec{r} = 1\hat{i} + \frac{1}{2}\hat{j} + 2\hat{k} + \lambda(2\hat{i} + 1\hat{k} + 3\hat{k})$

iv) $\dfrac{x}{2} = \dfrac{y}{5} = \dfrac{z}{3}$

Solution:- $\vec{r} = 0\hat{i} + 0\hat{j} + 0\hat{k} + \lambda(2\hat{i} + 5\hat{j} + 3\hat{k})$

Example:-18 Convert the vector form of line in symmetric form:-

i) $\vec{r} = 2\hat{i} + 3\hat{j} + 5\hat{k} + \lambda(\hat{i} + \hat{j} + 3\hat{k})$

Solution:- $\dfrac{x-2}{1} = \dfrac{y-3}{1} = \dfrac{z-5}{3}$

ii) $\vec{r} = 2\hat{i} - 3\hat{k} + \lambda(\hat{j} + \hat{k})$

Solution:- $\dfrac{x-2}{0} = \dfrac{y-0}{1} = \dfrac{z+3}{1}$

$x = 2, \quad \dfrac{y}{1} = \dfrac{z+3}{1}$

Example:- 19 Convert it in symmetric form:-

$2x+y=7, \quad z=4.$

Solution:- $2x = 7-y$

$$\frac{x}{1/2} = \frac{y-7}{-1} = \frac{z-4}{0}$$

Example:- 20 Find the equation of line passing through point $(2,2,3)$ and parallel to the vector $\vec{b} = \hat{i}+2\hat{j}+3\hat{k}$.

Sol^n:- $\vec{r} = \vec{a} + \lambda\vec{b}$

$\vec{r} \Rightarrow 2\hat{i}+2\hat{j}+3\hat{k} + \lambda(\hat{i}+2\hat{j}+3\hat{k})$

or,
$$\frac{x-2}{1} = \frac{y-2}{2} = \frac{z-3}{3}$$

Example:- 21 Find the equation of line in symmetric form which passes through $A(2,3,5)$, $B(5,7,9)$.

Solution:-
$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

$$\Rightarrow \frac{x-2}{5-2} = \frac{y-3}{7-3} = \frac{z-5}{9-5}$$

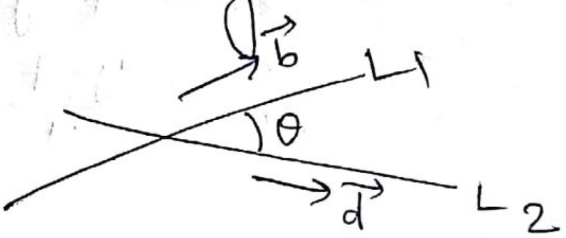
$$\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-5}{4}$$ Ans

4.3. Angle between two straight lines:-

Angle between two lines is same as the angle between the vectors along the line:

Let $L_1: \vec{r} = \vec{a} + \lambda\vec{b}$

$L_2: \vec{r} = \vec{c} + \mu\vec{d}$



then, $\vec{b}\cdot\vec{d} = |\vec{b}||\vec{d}|\cos\theta$

$$\boxed{\cos\theta = \frac{\vec{b}\cdot\vec{d}}{|\vec{b}||\vec{d}|}}$$

θ = angle between two line L_1 and L_2.

Example: 22 Find the angle between the line

$L_1: \vec{r} = (2\hat{i} + 3\hat{j} + 5\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + \hat{k})$

$L_2: \vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(\hat{i} + \hat{j} + \hat{k})$

Solution- Here, $\vec{b} = 3\hat{i} + 4\hat{j} + \hat{k}$

$\vec{d} = \hat{i} + \hat{j} + \hat{k}$.

$$\therefore \cos\theta = \frac{\vec{b}\cdot\vec{d}}{|\vec{b}||\vec{d}|} = \frac{3+4+1}{\sqrt{3^2+4^2+1^2}\sqrt{1^2+1^2+1^2}}$$

$$\cos\theta = \frac{8}{\sqrt{9+16+1}\sqrt{3}}$$

$$\cos\theta = \frac{8}{\sqrt{26\times3}} = \frac{8}{\sqrt{78}}$$

Angle b/w L_1 & L_2 = $\theta = \cos^{-1}\left(\frac{8}{\sqrt{78}}\right)$. Ans

By:- Niraj Keshri
B.Tech. IIT Kharagpur.

Example: 23 find the angle between the lines:-

$$L_1: \frac{x-1}{2} = \frac{y-2}{3} = \frac{z+3}{1}$$

$$L_2: \frac{x+5}{2} = \frac{y+7}{1} = \frac{z-9}{3}$$

Solution:- Here, $\vec{b} = 2\hat{i} + 3\hat{j} + 1\hat{k}$

$\vec{d} = 2\hat{i} + 1\hat{j} + 3\hat{k}$

$$\therefore \cos\theta = \frac{\vec{b}\cdot\vec{d}}{|\vec{b}||\vec{d}|} = \frac{4+3+3}{\sqrt{2^2+3^2+1^2}\sqrt{2^2+1^2+3^2}} = \frac{10}{\sqrt{4+9+1}\sqrt{4+1+9}}$$

$$= \frac{10}{14} = \frac{5}{7}$$

$$\boxed{\theta = \cos^{-1}(5/7)}$$ **Ans**

4.4. Condition of perpendicularity and parallelism:-

~~Case~~ $L_1: \vec{r} = \vec{a} + \lambda\vec{b}$

$L_2: \vec{r} = \vec{c} + \mu\vec{d}$

Say, $\vec{b} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$

$\vec{d} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$

$$\therefore \cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

* Condition of perpendicularity:-

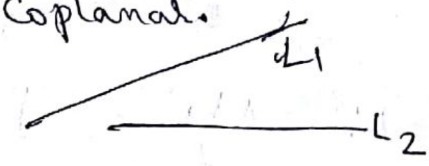
$\cos 90° = 0 \quad \therefore \boxed{a_1a_2 + b_1b_2 + c_1c_2 = 0}$

* Condition of parallelism:-

$\theta = 0°$, $\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}}$

4.5. Skew lines:-

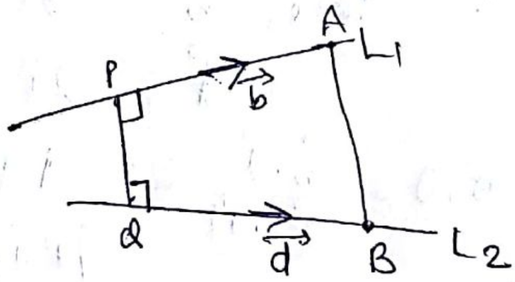
Two lines are said to be skew lines if they are neither parallel nor intersecting. Clearly, the skew lines can never be coplanar.



4.6. Shortest distance between two skew lines:-

Let $L_1 : \vec{r} = \vec{a} + \lambda \vec{b}$

$L_2 : \vec{r} = \vec{c} + \mu \vec{d}$



Let $OA = \vec{a}$

$OB = \vec{c}$

Here, $L_1 \parallel \vec{b}$ and $L_2 \parallel \vec{d}$

$\Rightarrow PQ \perp L_1$ and $PQ \perp L_2$

$\Rightarrow PQ \perp \vec{b}$ and $PQ \perp \vec{d}$

$\Rightarrow PQ \parallel \vec{b} \times \vec{d}$

Thus, shortest distance PQ = projection of AB on PQ.

$$= \frac{\vec{AB} \cdot \vec{PQ}}{|\vec{PQ}|}$$

*** $\boxed{\text{Shortest distance} = \frac{(\vec{a}-\vec{c}) \cdot (\vec{b} \times \vec{d})}{|\vec{b} \times \vec{d}|}}$

** for lines to be coplanar:-

$\boxed{(\vec{a}-\vec{c}) \cdot (\vec{b} \times \vec{d}) = 0}$

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4.7. Point of intersection of two lines:-

Example:-24 Find the point of intersection of the lines:-

L_1: $\frac{2x-1}{1} = \frac{y+3}{2} = \frac{z-2}{-1} = \lambda$

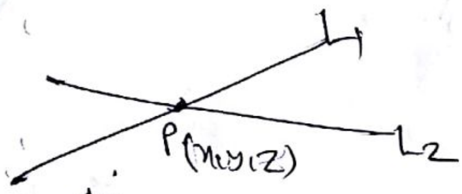
L_2: $\frac{x-1}{2} = \frac{y+5}{3} = \frac{z-1}{4} = \mu$

Solution:- General point on line L_1:-

$\left(\frac{1}{2}\lambda+\frac{1}{2},\ 2\lambda-3,\ -\lambda+2\right)$

General point on line L_2:-

$(2\mu+1,\ 3\mu-5,\ 4\mu+1)$



If two line intersection at a point then the x, y, and z coordinate at the point of intersection will be equal for both the lines:-

ie. $x_{L_1} = x_{L_2}$, $y_{L_1} = y_{L_2}$, $z_{L_1} = z_{L_2}$ at point of intersection

$\therefore \frac{\lambda}{2}+\frac{1}{2} = 2\mu+1$; $2\lambda-3 = 3\mu-5$, $-\lambda+2 = 4\mu+1$

$\frac{\lambda}{2} = 2\mu+\frac{1}{2}$; $2\lambda = 3\mu-2$

$\lambda = 4\mu+1$ —(1) ; $\lambda = \frac{3\mu-1}{2}$ —(2)

$\therefore 4\mu+1 = \frac{3\mu}{2}-1$

$2 = \frac{3\mu}{2} - 4\mu$

$2 = \frac{3\mu-8\mu}{2} = \frac{-5\mu}{2}$

$\boxed{\mu = -\frac{4}{5}}$

$\therefore \lambda = 4\times -\frac{4}{5}+1$

$= \frac{-16}{5}+1$

$\boxed{\lambda = \frac{-11}{5}}$

putting in z_{L_1} and z_{L_2} we get:-

$-\left(-\frac{11}{5}\right)+2 = \frac{11}{5}+2 = \frac{21}{5}$

$4\left(-\frac{4}{5}\right)+1 = -\frac{16}{5}+1 = \frac{-11}{5}$

$z_{L_1} \neq z_{L_2}$

Hence, there are no any point where line L_1 and L_2 intersect.

Therefore L_1 and L_2 are skew lines.

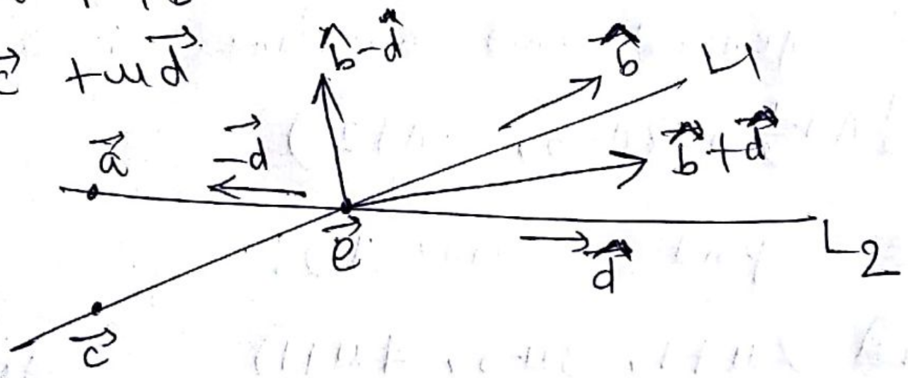
* Equation of angle bisector:-

Example:- 25 Find the angle bisector of line:-

$L_1 \to \vec{r} = \vec{a} + \lambda\vec{b}$

$L_2 :- \vec{r} = \vec{c} + \mu\vec{d}$

Solution:-



Angle bisector of L_1 and L_2 will lie along the $(\hat{b}+\hat{d})$ and $(\hat{b}-\hat{d})$ vectors.

We will first find the point of intersection of two lines L_1 and L_2 which comes to be say $\vec{e}$.

then, we can write the equation of line passing through $\vec{e}$ and parallel to $(\hat{b}+\hat{d})$ or $\hat{b}-\hat{d}$. That will be the equation of angle bisectors.

$$\left.\begin{aligned} \vec{r} &= \vec{e} + \lambda(\hat{b}+\hat{d}) \\ \vec{r} &= \vec{e} + \mu(\hat{b}-\hat{d}) \end{aligned}\right\} \text{ are angle bisectors of } L_1 \text{ and } L_2.$$

By: Niraj Keshri
B. Tech. IIT Kharagpur

The plane

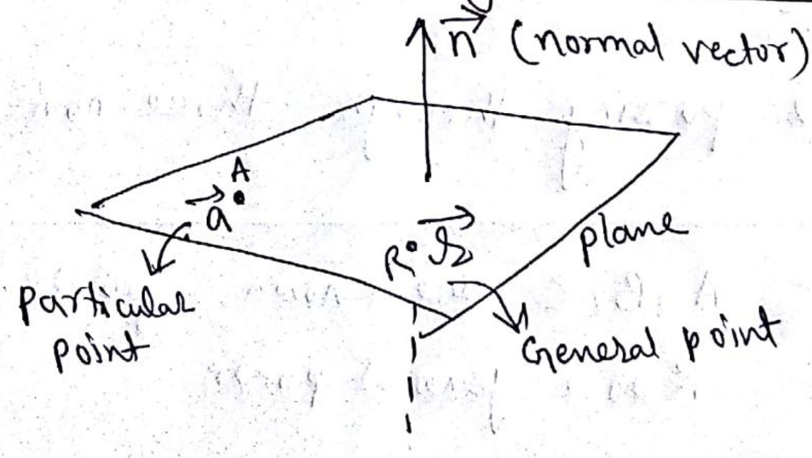
5.1. General form of the plane

A first degree equation represents a plane. The general equation of a plane is given by:-

$$\boxed{ax + by + cz + d = 0}$$

A plane is a surface such that if any two points on it are taken, the line joining them completely lies on it.

5.2. Equation of the plane having normal vector given and passing through a point.



$\vec{n}$ = normal vector

$\vec{n} = a\hat{i} + b\hat{j} + c\hat{k}$

$\vec{a}$ = a particular point (Known point)

$\vec{r}$ = General point $(x\hat{i} + y\hat{j} + z\hat{k})$

We can see that,

$\vec{AR}$ is perpendicular to $\vec{n}$.

$\therefore \vec{AR} \cdot \vec{n} = 0$

$(\vec{r} - \vec{a}) \cdot \vec{n} = 0 \Rightarrow \boxed{\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}}$

$\vec{r} \cdot \vec{n} - \vec{a} \cdot \vec{n} = 0$

Vector form of a plane.

In Cartesian form:-

$\vec{r}\cdot\vec{n} = \vec{a}\cdot\vec{n}$

$\vec{a} = x_0\hat{i} + y_0\hat{j} + z_0\hat{k}$

$\vec{n} = a\hat{i} + b\hat{j} + c\hat{k}$

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$(x\hat{i}+y\hat{j}+z\hat{k})\cdot(a\hat{i}+b\hat{j}+c\hat{k}) = (x_0\hat{i}+y_0\hat{j}+z_0\hat{k})\cdot(a\hat{i}+b\hat{j}+c\hat{k})$

$\Rightarrow ax+by+cz = \underbrace{(ax_0+by_0+cz_0)}_{\text{Constant value } (K)}$

$\Rightarrow ax+by+cz-K=0$

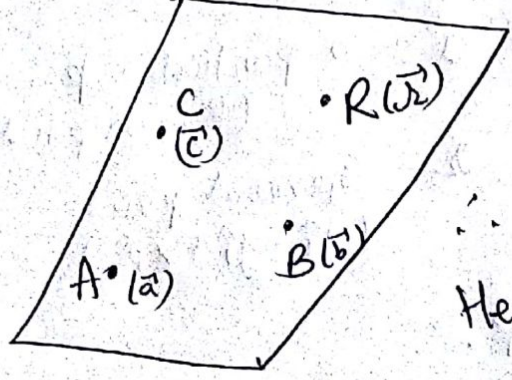
$\Rightarrow \boxed{ax+by+cz+d=0}$ $\boxed{d=-K}$ another Constant

Cartesian equation of plane

or $\boxed{a(x-x_0)+b(y-y_0)+c(z-z_0)=0}$

Equation of plane:-

5.2 Equation of plane passing through three non-Collinear points:-



A, B, C are known points.

R is a general point.

$\therefore \vec{AB}, \vec{AC}, \vec{AR}$ are Coplanar

Hence, $[\vec{AB}, \vec{AC}\ \vec{AR}]=0$

$\Rightarrow [\vec{b}-\vec{a},\ \vec{c}-\vec{a},\ \vec{r}-\vec{a}]=0$

By:- Niraj Keshri
B.Tech. IIT Kharagpur

6. If $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ $\quad A(x_1, y_1, z_1)$

$\vec{b} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$ $\quad B(x_2, y_2, z_2)$

$\vec{c} = x_3\hat{i} + y_3\hat{j} + z_3\hat{k}$ $\quad C(x_3, y_3, z_3)$

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ $\quad R(x, y, z)$

$\therefore$

$$[\vec{b}-\vec{a},\ \vec{c}-\vec{a},\ \vec{r}-\vec{a}] = 0$$

$$\boxed{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \\ x-x_1 & y-y_1 & z-z_1 \end{vmatrix} = 0}$$

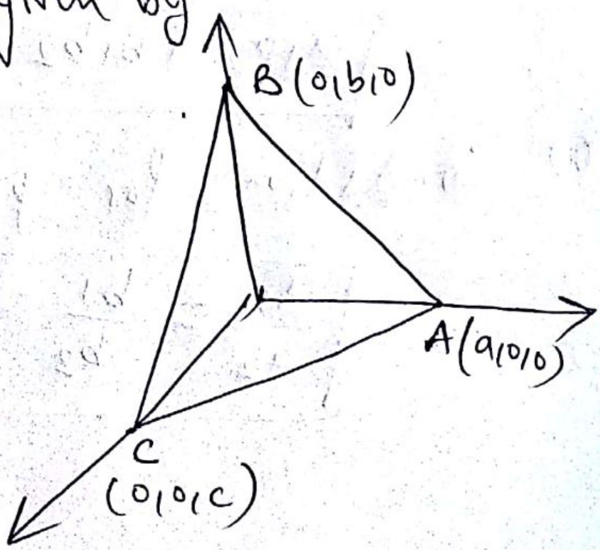
Equation of plane

5.4. Intercept form of a plane:-

If a plane makes an intercept of a, b, c on the coordinate axes, then equation of the plane is given by

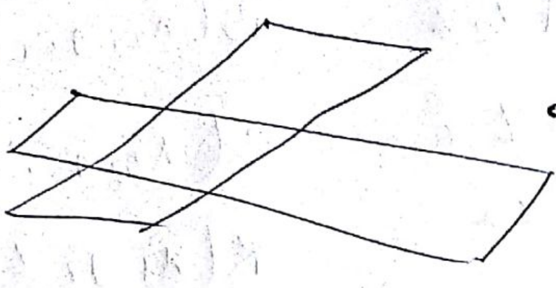
$\cancel{\frac{x}{a}} + \cancel{\frac{y}{b}} + \cancel{z}$

$$\boxed{\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1}$$

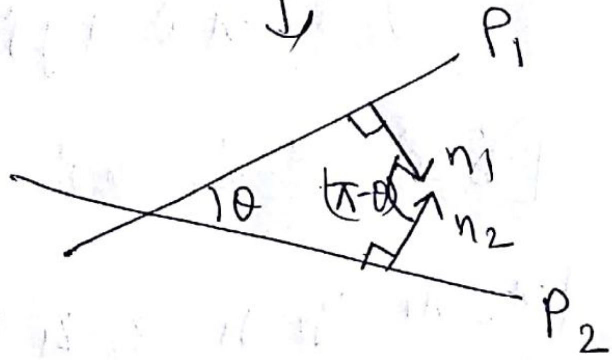


5.5. Angle between two planes:-

The angle between two planes is defined as the angle between their normals.



→ 2D view



$$\cos(\pi-\theta) = \frac{\vec{n_1}\cdot\vec{n_2}}{|\vec{n_1}|\,|\vec{n_2}|}$$

$$\boxed{\cos\theta = \frac{\vec{n_1}\cdot\vec{n_2}}{|\vec{n_1}|\,|\vec{n_2}|}}$$

θ = Angle b/w the planes.

5.6. Condition for perpendicularity and parallelism:-

$\vec{n_1} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$

$\vec{n_2} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$

* Condition for perpendicularity:-

$$\boxed{a_1a_2 + b_1b_2 + c_1c_2 = 0}$$

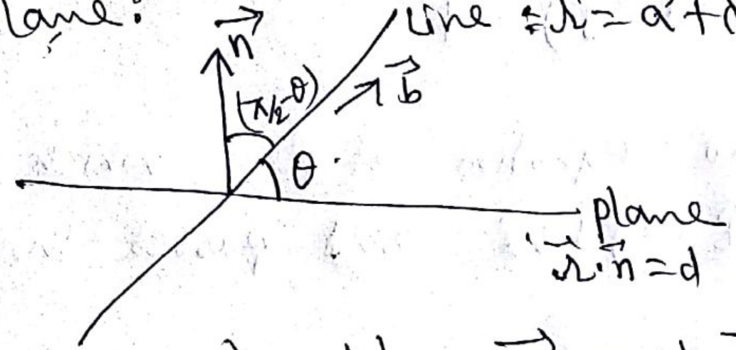
* Condition for parallelism:-

$$\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}}$$

By: Niraj Keshri
B.Tech. IIT Kharagpur

5.7. Angle between a line and a plane:-

The angle between a line and a plane is the angle between the line and the normal to the plane:



(2-D-View)

$\therefore$ angle b/w $\vec{n}$ and $\vec{b}$ is $(\pi/2 - \theta)$

$$\therefore \cos\left(\frac{\pi}{2} - \theta\right) = \frac{\vec{n}\cdot\vec{b}}{|\vec{n}||\vec{b}|}$$

$$\boxed{\sin\theta = \frac{\vec{n}\cdot\vec{b}}{|\vec{n}||\vec{b}|}}$$

5.8. Condition of perpendicularity and parallelism:-

Let $\vec{n} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$

$\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$

* Condition of perpendicularity:- $(\theta = \pi/2)$

$\sin\pi/2 = 1$

$$\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}}$$

* Condition of parallelism:- $\theta = 0$, $\sin\theta = 0$

$$\boxed{a_1a_2 + b_1b_2 + c_1c_2 = 0}$$

Example:- 26 Convert $\vec{r}\cdot(\hat{i}+2\hat{j}-\hat{k})+4=0$ in Cartesian form

Solution:- $x+2y-z+4=0$ Ans

Example:- 27 Convert $2x+3y=7$ in vector form.

Solution:- $\vec{r}\cdot(2\hat{i}+3\hat{j})=7$

Example:- 28 Find the equation of plane which is perpendicular to $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$ and passes through $(2,5,7)$.

Solution:- Since $\vec{a}$ is $\perp$r to plane, $\vec{a}$ is a normal vector to the plane. therefore the equation of plane will be:- ~~$1(x-2)+2(y-5)+3(x-$~~

$$1(x-2)+2(y-5)+3(z-7)=0$$ Ans

Example:- 29 Find the equation of plane which passes through $A(2,3,4)$, $B(3,4,5)$, $C(9,5,6)$.

Solution:- Equation of plane will be given by:-

$$\begin{vmatrix} x-2 & y-3 & z-4 \\ 3-2 & 4-3 & 5-4 \\ 9-2 & 5-3 & 6-4 \end{vmatrix}=0$$

$$\begin{vmatrix} x-2 & y-3 & z-4 \\ 1 & 1 & 1 \\ 7 & 2 & 2 \end{vmatrix}=0$$

Example:- 30 If the line L: $\frac{x-1}{2} = \frac{2y-3}{4} = \frac{z+4}{-1}$ is perpendicular to the plane passing through (1, 3, 4). find the plane.

Solution:- Since line is perpendicular to the plane therefore $\vec{b}$ of the line will be the normal vector for the ~~fo~~ plane. $\vec{b} = 2\hat{i} + 2\hat{j} - \hat{k}$.
Hence equation of plane will be

$$2(x-1) + 2(y-3) + -1(z-4) = 0$$ Ans

Example:- 31 Find the angle between two planes:-

$P_1: x + y + z + 3 = 0$

$P_2: 5x + 5y + 5z + 19 = 0$

Solution:- Since we can see from the equation of planes that $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, ~~that~~ therefore from the condition of parallelism we can say that both planes are parallel.

Example:- 32 ~~find angle~~ Find 'K' If two plane are perpendicular: $P_1: x + 2y + 3z + 9 = 0$

$P_2: k + 3y + 7z + 11 = 0$

Solution:- If two plane are ~~per~~ $\perp r$, then

$a_1a_2 + b_1b_2 + c_1c_2 = 0 \Rightarrow \boxed{K = -27}$ Ans

$\therefore 1 \cdot k + 2 \cdot 3 + 3 \cdot 7 = 0$

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Q. **Example :- 33** Find the angle between the line

$L: \frac{x-1}{3} = \frac{y+3}{4} = \frac{z}{3}$ and plane $P_1: 2x+3y+z+9=0$

Solution :- angle b/w line and plane is given by

$$\sin\theta = \frac{\vec{n}\cdot\vec{b}}{|\vec{n}||\vec{b}|}$$

Here, $\vec{n} = 2\hat{i}+3\hat{j}+\hat{k}$

$\vec{b} = 3\hat{i}+4\hat{j}+3\hat{k}$

$$\therefore \sin\theta = \frac{2\cdot3+3\cdot4+1\cdot3}{\sqrt{2^2+3^2+1^2}\sqrt{3^2+4^2+3^2}}$$

$$= \frac{6+12+3}{\sqrt{4+9+1}\sqrt{9+16+9}} = \frac{21}{\sqrt{14\times34}}$$

$$\boxed{\theta = \sin^{-1}\left(\frac{21}{\sqrt{14\times34}}\right)}$$ Ans

Example :- 34 Find the equation of plane which makes equal intercept on the axes and passes through $(1,1,1)$.

Solution :- Equation of plane in intercept form is

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \quad [\text{given } a=b=c]$$

$\therefore x+y+z=a$ is

$(1,1,1)$ satisfy the equation

$\therefore 1+1+1=a \quad \therefore a=3$

Hence, $\boxed{x+y+z=3}$ is the equation of plane

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B.Tech. IIT Kharagpur

5.9. Equation of a plane parallel to a given plane:-

The equation of a plane parallel to a given plane $ax+by+cz+d=0$ is given by:-

$$\boxed{ax+by+cz+K=0}$$

Example:- 35 Find the equation of plane passing through $(1,2,3)$ and parallel to ~~2x~~ the plane $2x+3y+5z+9=0$ is:-

Solution:- Equation of plane parallel to $2x+3y+5z+9=0$ will be given by

$2x+3y+5z+K=0$, and since it passes through $(1,2,3)$, ~~&~~ we will have

$$2\cdot1+3\cdot2+5\cdot3+K=0$$

$$2+6+15+K=0$$

$$\boxed{K=-23}$$

$\therefore$ Equation of plane is $\boxed{2x+3y+5z-23=0}$

Ans

5.10. Equation of a plane parallel to the axes:-

- **Equation of a plane parallel to x-axis:-**

Let the equation of plane be $ax+by+cz+d=0$,
and the equation of x-axis is:

$$\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$$

Since the plane is parallel to the line, so

$$a \cdot 1 + b \cdot 0 + c \cdot 0 = 0$$

$$\boxed{a=0}$$

Hence, The plane parallel to x axis is given by

$$\boxed{by+cz+d=0}$$

- Similarly, Equation of plane parallel to y-axis is given by $\boxed{ax+cz+d=0}$

and Equation of plane parallel to z-axis is given as:- $\boxed{ax+by+d=0}$

- **Equation of plane parallel to xy-plane:-**

The equation of plane xy is given by

$$\boxed{z=0}$$

∴ The plane parallel to xy-plane will be given by $\boxed{z+k=0}$

Similarly, The plane parallel to yz-plane and zx-plane is given by $\boxed{x+k=0}$ and $\boxed{y+k=0}$ respectively.

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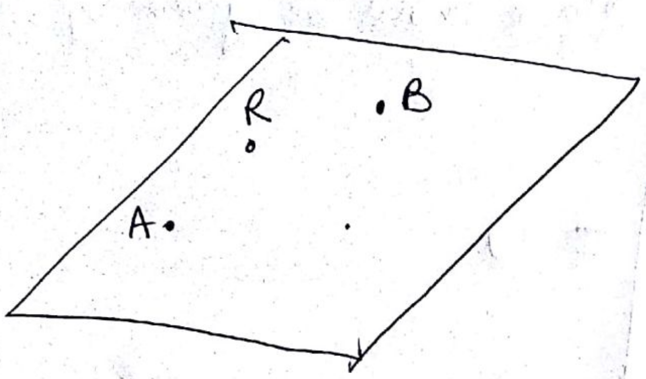
5.11. Equation of a plane passing through two given points and parallel to the line having direction ratios a, b, c.



$A(x_1, y_1, z_1)$

$B(x_2, y_2, z_2)$

$R(x, y, z)$

$\vec{c} = a\hat{i} + b\hat{j} + c\hat{k}$

Since, $\vec{AB}, \vec{AR}, \vec{c}$ are co-planar, we have

$$[\vec{AB}, \vec{AR}, \vec{c}] = 0$$

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a & b & c \\ x - x_1 & y - y_1 & z - z_1 \end{vmatrix} = 0$$

or

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a & b & c \end{vmatrix} = 0$$

is the required equation of the plane

Example:- 36 Find the equation of the plane passing through A(2,3,4), B(4,5,6) and parallel to a line having direction ratio (1, 2, 3).

Solution:- The equation of plane will be given by:-

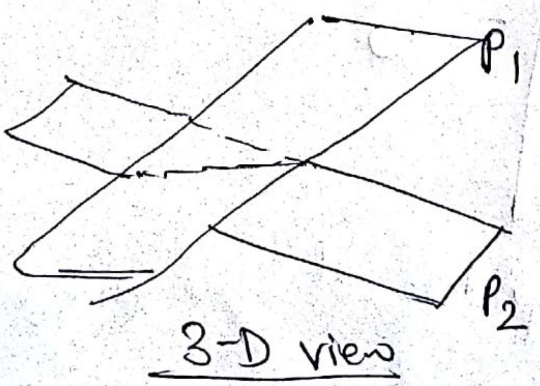
$$\begin{vmatrix} x-2 & y-3 & z-4 \\ 4-2 & 5-3 & 6-4 \\ 1 & 2 & 3 \end{vmatrix} = 0$$

or

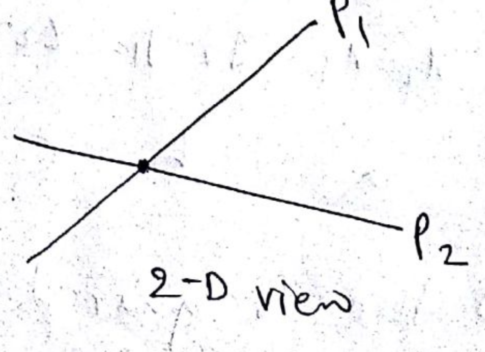
$$\begin{vmatrix} x-2 & y-3 & z-4 \\ 2 & 2 & 2 \\ 1 & 2 & 3 \end{vmatrix} = 0$$

5.12. Equation of the plane passing through the line of intersection of planes:-

The is also called family of planes:-



3-D view



2-D view

There can be infinite no. of planes: which passes through the line of intersection of two planes.

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B. Tech, IIT Kharagpur

These planes are called family of planes and their equation is given by:-

$\boxed{P_1 + \lambda P_2 = 0}$ λ is an arbitrary parameter

If the equation of $P_1 = a_1x + b_1y + c_1z + d_1 = 0$

$P_2 = a_2x + b_2y + c_2z + d_2 = 0$

then equation of family of plane will be

$$(a_1x + b_1y + c_1z + d_1) + \lambda(a_2x + b_2y + c_2z + d_2) = 0$$

Example:- 37 Find the equation of the plane passing through the line of intersection of planes P_1: $2x + 3y + 4z + 9 = 0$ and
P_2: $x + y + 3z + 7 = 0$ and also passing through point $(1, 1, 1)$.

Solution:- Equation of plane passing through line of intersection of two plane is given by:-

$$\Rightarrow (2x + 3y + 4z + 9) + \lambda(x + y + 3z + 7) = 0$$

It passes through $(1, 1, 1)$ therefore

$$(2\cdot1 + 3\cdot1 + 4\cdot1 + 9) + \lambda(1\cdot1 + 1\cdot1 + 3\cdot1 + 7) = 0$$

$$18 + \lambda(12) = 0$$

$\therefore 12\lambda = -18$

$\lambda = \frac{-\cancel{18}\,3}{\cancel{12}\,2} = -3/2$

Hence the required equation of the plane will be: $(2x+3y+4z+9) - \frac{3}{2}(x+y+3z+7) = 0$

$\left(2-\frac{3}{2}\right)x + \left(3-\frac{3}{2}\right)y + \left(4-\frac{9}{2}\right)z + 9 - \frac{21}{2} = 0$

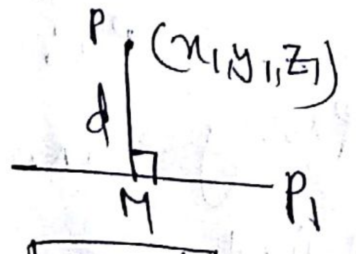
$\frac{x}{2} + \frac{3}{2}y - \frac{z}{2} - \frac{3}{2} = 0$

$\boxed{x+3y-z-3=0}$ Ans

5.13. Perpendicular distance of a point from a plane

The length of $\perp r$ from a point $P(x_1, y_1, z_1)$ to the plane $ax+by+cz+d=0$, is given by

$$d = \left| \frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}} \right|$$



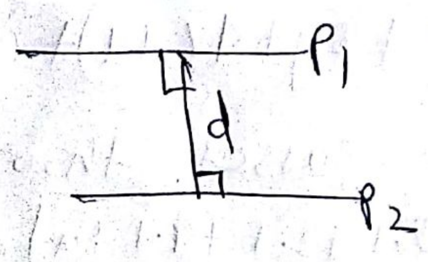
$\boxed{PM = d}$

5.14. Distance between two parallel planes:

P_1: $a_1x+b_1y+c_1z+d_1=0$

P_2: $a_1x+b_1y+c_1z+d_2=0$

$$\therefore \boxed{d = \left| \frac{d_2-d_1}{\sqrt{a_1^2+b_1^2+c_1^2}} \right|}$$



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5.15. Location of a point w.r.t. a plane:-

We know that the plane divides the three-dimensional space into two equal parts. So o points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ are on the same side or opposite side of the plane $ax+by+cz+d=0$ if

$$\frac{ax_1+by_1+cz_1+d}{ax_2+by_2+cz_2+d} > 0 \text{ or } < 0.$$

5.16. Intersection of a line and a plane:-

Let:- $L: \frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$.

$P: Ax+By+Cz+D=0$

Any point on the line L can be considered as $(x_1+a\lambda, y_1+b\lambda, z_1+c\lambda)$.

If it lies on the plane 'P' then

$$A(x_1+a\lambda)+B(y_1+b\lambda)+C(z_1+c\lambda)+D=0$$

$$\Rightarrow (Ax_1+By_1+Cz_1+D)+\lambda(Aa+Bb+Cc)=0$$

$$\Rightarrow \lambda = -\frac{(Ax_1+By_1+Cz_1+D)}{(Aa+Bb+Cc)}$$

Substituting the value of λ in general point considered, we get the required co-ordinates of the point of intersection.

<u>Example:- 38</u> Find the point of intersection of the line $\frac{x-1}{1}=\frac{y-2}{3}=\frac{z}{4}$ and plane $2x+3y+4z+7=0$.

<u>Solution:-</u> Any point on line is given by

$$(\lambda+1,\ 3\lambda+2,\ 4\lambda)$$

This lies on the plane,

$$\therefore\ 2(\lambda+1)+3(3\lambda+2)+4(4\lambda)+7=0$$

$$\Rightarrow 2\lambda+2+9\lambda+6+16\lambda+7=0$$

$$\Rightarrow 27\lambda=-15$$

$$\boxed{\lambda=\frac{-15}{27}}$$

$\therefore$ point of intersection will be :-

$$\left(\frac{-15}{27}+1,\ 3\left(\frac{-15}{27}\right)+2,\ 4\left(\frac{-15}{27}\right)\right)$$

$$\left(\frac{\cancel{12}^{4}}{\cancel{27}_{9}},\ \frac{\cancel{9}^{1}}{\cancel{27}_{3}},\ \frac{-\cancel{60}^{20}}{\cancel{27}_{9}}\right)$$

$$\left(\frac{4}{9},\ \frac{1}{3},\ -\frac{20}{9}\right)\ \text{Ans}$$

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5.17. Condition for a line to lie on a plane:-

If the line $L: \frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$.

lies in the plane $P: ax+by+cz+d=0$, then

i) $ax_1+by_1+cz_1+d=0$

ii) $al+bm+cn=0$.

Since, the line $L=0$ lies in a plane $P=0$, therefore the point (x_1, y_1, z_1) on L will also lie in the plane $P=0$,

So, $\boxed{ax_1+by_1+cz_1+d=0}$

Also, Since the line $L=0$ lies in the plane $P=0$, therefore, the normal to the plane is also normal to the line.

Thus, $\boxed{al+bm+cn=0}$.

Example-39 Check if $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{1}$ lies on plane $x+y+z+3=0$.

Solution

on putting $(1,1,1)$ on plane, we get

$1+1+1+3 \neq 0$, Hence

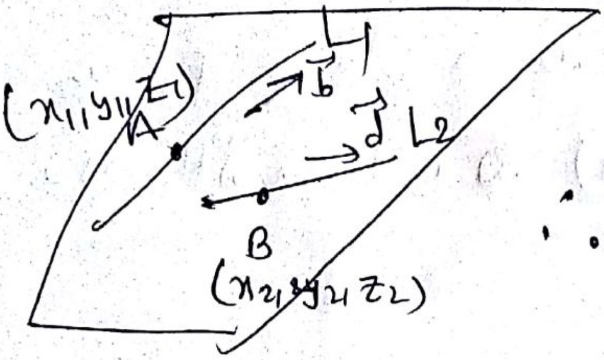
Line doesn't lie on plane.

5.18 Condition of co-planarity of two lines:-

Let L_1: $\dfrac{x-x_1}{a_1} = \dfrac{y-y_1}{b_1} = \dfrac{z-z_1}{c_1}$

L_2: $\dfrac{x-x_2}{a_2} = \dfrac{y-y_2}{b_2} = \dfrac{z-z_2}{c_2}$

If two lines L_1 and L_2 are Coplanar, then



then $\vec{b}, \vec{d}, \vec{AB}$ are coplanar

$\therefore$ $\vec{b} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$

$\vec{d} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$

$\vec{AB} = (x_2-x_1)\hat{i} + (y_2-y_1)\hat{j} + (z_2-z_1)\hat{k}$

$\therefore$ $[\vec{AB}, \vec{b}, \vec{d}] = 0$

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

and equation of plane containing them will be:-

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0 \quad \text{or} \quad \begin{vmatrix} x-x_2 & y-y_2 & z-z_2 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

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5.19/ 5.20/ Equation of the planes bisecting the angle between the planes:-

Let equation of the two planes be:

$a_1x + b_1y + c_1z + d_1 = 0$

$a_2x + b_2y + c_2z + d_2 = 0$

First we make $d_1 > 0$, $d_2 > 0$.

(i) If $a_1a_2 + b_1b_2 + c_1c_2 > 0$, the origin lies in the obtuse angle between two planes and the equation of the obtuse angle bisector is

$$\frac{a_1x + b_1y + c_1z + d_1}{\sqrt{a_1^2 + b_1^2 + c_1^2}} = + \frac{a_2x + b_2y + c_2z + d_2}{\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

and the acute angle bisector is

$$\frac{a_1x + b_1y + c_1z + d_1}{\sqrt{a_1^2 + b_1^2 + c_1^2}} = - \frac{a_2x + b_2y + c_2z + d_2}{\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

(ii) If $a_1a_2 + b_1b_2 + c_1c_2 < 0$, the origin lies in the acute angle between the two planes and the equation of the bisector of the acute angle is:-

$$\frac{a_1x+b_1y+c_1z+d_1}{\sqrt{a_1^2+b_1^2+c_1^2}} = + \frac{a_2x+b_2y+c_2z+d_2}{\sqrt{a_2^2+b_2^2+c_2^2}}$$

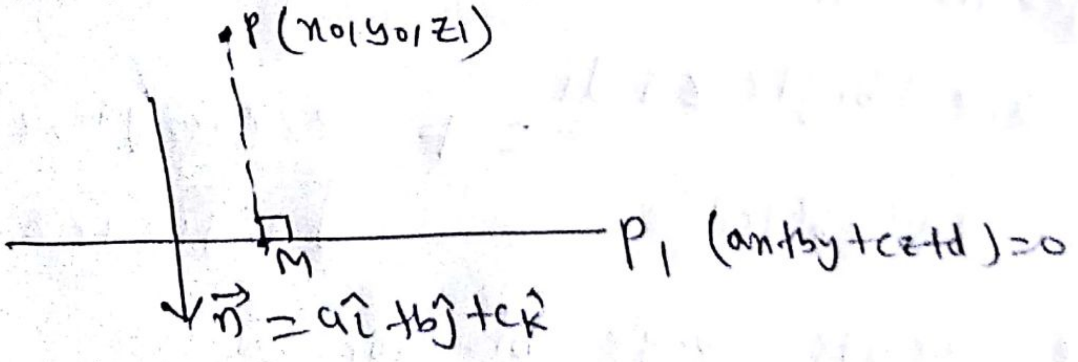
and the obtuse angle bisector is:-

$$\frac{a_1x+b_1y+c_1z+d_1}{\sqrt{a_1^2+b_1^2+c_1^2}} = - \frac{a_2x+b_2y+c_2z+d_2}{\sqrt{a_2^2+b_2^2+c_2^2}}$$

5.21. Foot of perpendicular of a point w.r.t. a plane:-

Let plane be:- $P: ax+by+cz+d=0$

And point $P(x_0, y_0, z_0)$



Equation of line PM:- $\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$,

Considering general point on PM line

$(x_0+a\lambda,\ y_0+b\lambda,\ z_0+c\lambda)$. This lies on plane $P_1\ (ax+by+cz+d=0)$ is M is foot of ⊥r.

$\therefore P \Rightarrow a(x_0+a\lambda)+b(y_0+b\lambda)+c(z_0+c\lambda)+d=0$

we got $\lambda = \frac{-(ax_0+by_0+cz_0+d)}{a^2+b^2+c^2}$

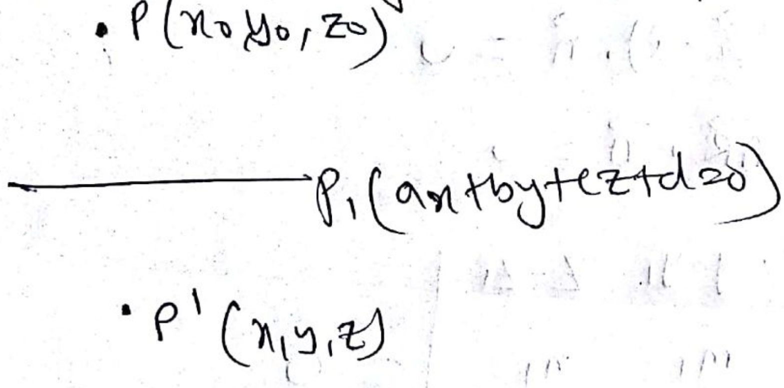
∴ we can get the point M:-

~~the Hence, equation of foo~~

∴ foot of perpendicular is given by:-

$$\boxed{\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c} = \frac{-(ax_0+by_0+cz_0+d)}{a^2+b^2+c^2}}$$

5.22. Image of a point w.r.t. a plane:-



Coordinates of Image can be found using the given formula:-

$$\boxed{\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c} = -2\frac{(ax_0+by_0+cz_0+d)}{a^2+b^2+c^2}}$$

5.33. Equation of a plane containing a given line and parallel to a given line:-

The equation of plane containing a given line

L_1: $\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$, and also parallel

to another line L_2: $\frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$

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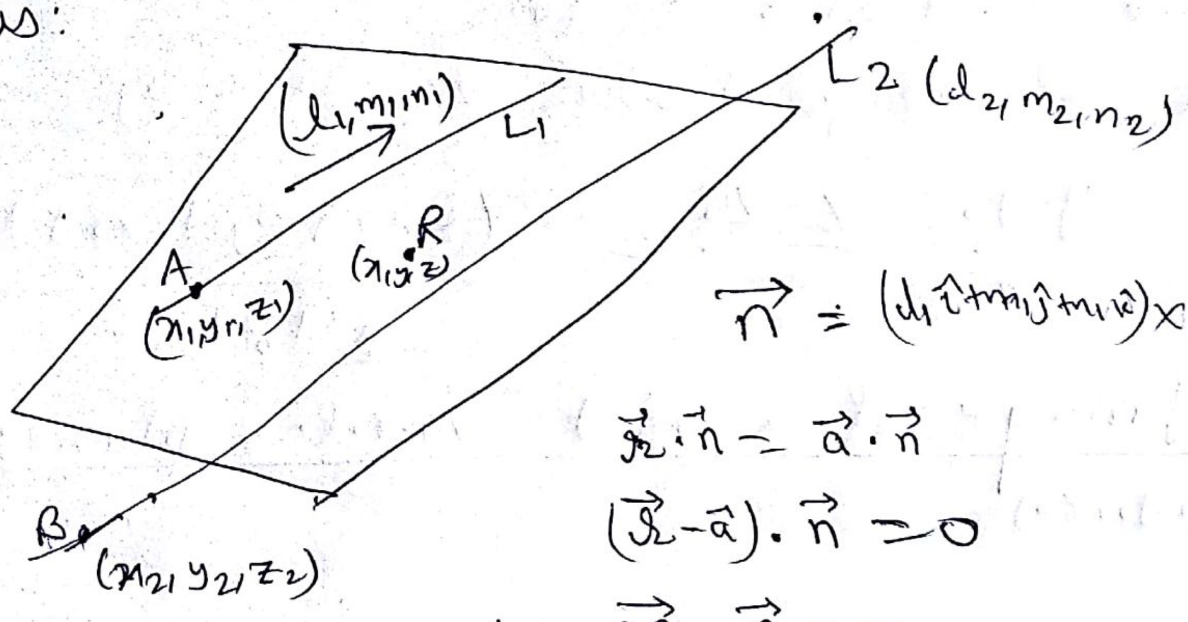
is:



$$\vec{n} = (l_1\hat{i}+m_1\hat{j}+n_1\hat{k}) \times (l_2\hat{i}+m_2\hat{j}+n_2\hat{k})$$

$$\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$$

$$(\vec{r}-\vec{a}) \cdot \vec{n} = 0$$

$$\therefore \ \vec{AR} \cdot \vec{n} = 0$$

$$\Rightarrow \begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

Normal vector of the plane will lie along the $(\vec{b} \times \vec{d})$.
Hence, taking dot product with $\vec{AR}$ will give the result.

5.24. Equation of the plane containing two given lines:-

~~Let~~:- equation of the plane ~~Contaig~~ Containing two lies are given in 5.18. Section.. Coplanarity of two lines.

—— x ——

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B. Tech. IIT Kharagpur

Question Bank

Q.1 The shortest distance ~~bo~~ of the point (a,b,c) from the x-axis is:-

A) $\sqrt{a^2+b^2}$ B) $\sqrt{a^2+c^2}$ C) $\sqrt{b^2+c^2}$ D) $\sqrt{a^2+b^2+c^2}$

Q.2 The equation of the x-axis is:-

A) $\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$ B) $\frac{x}{0} = \frac{y}{1} = \frac{z}{0}$

C) $\frac{x}{0} = \frac{y}{0} = \frac{z}{1}$ C) $\frac{x}{1} = \frac{y}{0} = \frac{z}{1}$

Q.3 If the direction cosines of a line are $\left(\frac{1}{c}, \frac{1}{c}, \frac{1}{c}\right)$, then c is :-

A) 1 B) ± 2 C) ± 3 D) $\pm\sqrt{3}$

Q.4. The number of lines, through origin makes equal angle with the axes, is

A) 1 B) 2 C) 4 D) 0

Q.5 The foot of perpendicular from (a,b,c) on the z-axis is:-

A) (a,0,0) B) (0,b,0) C) (0,0,0) D) (0,0,c)

Q.6. Angle between any two diagonals of a cube is:-

A) $\tan^{-1}(2\sqrt{2})$ B) $\tan^{-1}(\sqrt{2})$

C) $\cot^{-1}(\sqrt{2})$ D) $\cos^{-1}(2/3)$

Q.7 The angle between a diagonal of a unit cube and a diagonal of a face is :-

A) $\cos^{-1}(2/3)$ B) $\cos^{-1}(\sqrt{2/3})$

C) $\cos^{-1}(\sqrt{1/3})$ D) $\cos^{-1}(1/3)$

Q.8 A line makes an angle α, β and γ with the axes, then the value of

$\cos 2\alpha + \cos 2\beta + \cos 2\gamma$ is :-

A) 1 B) 2 C) −1 D) 0

Q.9 The shortest distance ~~be~~ from origin to the plane $2x + 3y + 6z - 21 = 0$ is :-

A) 2 B) 1 C) 3 D) 4.

Q.10 The locus of the first degree equation in x, y, z represts a

A) Straight line B) plane C) Sphere D) Conicoid.

Q.11 If the plane $2x - 3y + 6z - 11 = 0$ makes an angle $\sin^{-1}(K)$ with x-axis, then K is :-

A) $\sqrt{3}/2$ B) $2/7$ C) $\sqrt{2}/3$ D) 1.

Q.12 The distance between the parallel planes $2x - 3y + 6z + 5 = 0$ and $6x - 9y + 18z + 21 = 0$ is :-

A) $1/7$ B) $3/7$ C) $2/7$ D) $4/7$.

Q.13 The equation of the plane passing through (1,2,3) and parallel to $2x + y + 2z + 10 = 0$ is.

A) $2x + y + 2z - 10 = 0$ B) $2x + y + 2z - 12 = 0$

C) $2x + y + 2z + 12 = 0$ D) $2x + y + 2z + 10 = 0$

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Q.14 The angle between the planes $2x-y+z=6$ and $x+y+2z=7$ is:-

A) $\pi/4$ B) $\pi/3$ C) $\pi/6$ D) $\pi/2$

Q.15 The line $x=1, y=2$ is:-

A) parallel to x-axis B) parallel to y-axis

C) parallel to z-axis D) none of these.

Q.16 The equation of the ~~plan~~ line passing through $(2,3,4)$ and perpendicular to the plane $2x+3y-z=5$ is:-

A) $\frac{x-2}{2} = \frac{y-3}{1} = \frac{z-4}{3}$ B) $\frac{x-2}{2} = \frac{y+3}{1} = \frac{z-4}{3}$

C) $\frac{x-2}{2} = \frac{y+3}{1} = \frac{z-4}{3}$ D) $\frac{x-2}{2} = \frac{y-3}{4} = \frac{z-4}{5}$

Q.17 The line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ is parallel to the plane:-

A) $3x+4y+5z=7$ B) $2x+3y+5z=10$

C) $2x+y-2z=0$ D) $2x+3y+2z=5$

Q.18 The angle between the plane $3x+6y-2z+5=0$ and the line $\frac{x-2}{2} = \frac{y+1}{-1} = \frac{z-3}{2}$ is

A) $\cos^{-1}\left(\frac{4}{21}\right)$ B) $\cos^{-1}(8/21)$ C) $\sin^{-1}(4/21)$

D) $\sin^{-1}(8/21)$

Q.19 The co-ordinates of the point of intersection of the line $\frac{x-6}{-1} = \frac{y+1}{0} = \frac{z+3}{4}$ and the plane $x+y-z=3$ is:-

A) $(2,1,0)$ B) $(1,2,-6)$ C) $(5,1,2)$ D) $(5,-1,1)$

Q.20 The Shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is:-

A) $1/6$ B) $1/\sqrt{6}$ C) $1/\sqrt{3}$ D) $1/3$

Q.21 The plane passing through $(3,2,0)$ and the line $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$ is:-

A) $x-y+z=1$ B) $x+y+z=5$ C) $x+2y-z=1$

D) $2x-y+z=5$

Q.22. The distance of the plane passing through $(1,1,1)$ and perpendicular to the line $\frac{x-1}{3} = \frac{y-1}{0} = \frac{z-1}{4}$ from the origin is:-

A) $3/4$ B) $4/3$ C) $7/5$ D) 1

Q.23 The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{4-z}{k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar if k is:-

A) 1 B) 2 C) 0 D) 4

By:- Niraj Keshri
B.Tech. IIT Kharagpur

Q.24 The equation of the obtuse angle bisector between the planes $3x-2y+6z+8=0$ and $2x-y+2z+3=0$ is:-

A) $5x-y-4z=3$ B) $5x+y-4z=5$

C) $4x+y-5z=5$ D) $7x+3y-9z=0$

Q.25 The plane $x-y-z=4$ is rotated through an angle $90°$ about its line of intersection with the plane $x+y+2z=4$. Then the new position of the plane is:-

A) $5x+3y+2z=4$ B) $5x+y+4z=20$

C) $4x+y+5z=20$ D) $4x+5y+z=20$

Q.26 The value of m for which the straight lines $3x-2y+z+3=0=4x-3y+4z+1$ are parallel to the plane $2x-y+mz=2$ is:-

A) 6 B) 8 C) -2 D) -11

Q.27 Let $L_1: \frac{x-2}{1}=\frac{y-1}{0}=\frac{z+1}{2}$; $L_2: \frac{x-3}{1}=\frac{y-1}{1}=\frac{z}{-1}$ and let π be the plane which contains the line L_1 and is parallel to L_2. The distance of the plane π from origin is:-

A) $\sqrt{2/7}$ B) $1/7$ C) $\sqrt{6}$ D) None.

Q.28 If the line $\frac{x-2}{3} = \frac{1-y}{6} = \frac{z+2}{2}$ lies in the plane $x+3y-az+b=0$. Then the value of $(a+b+1)$ is:-

A) 2 B) 1 C) 3 D) 4.

Q.29 If the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$; $\frac{x-1}{3} = \frac{y-2}{-1} = \frac{z-3}{4}$ and $\frac{x+k}{3} = \frac{y-1}{2} = \frac{z-2}{h}$ are concurrent, then

A) $h=-2$, $k=-6$ B) $h=6$, $k=2$

C) $h=1/2$, $k=2$ D) $h=2$, $k=1/2$

Q.30 ~~Find~~ If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect at a point, the value of k is:-

A) $3/2$ B) $9/2$ C) $2/9$ D) $-3/2$

Q.31 The equation of a plane passing through the line of intersection of the planes $x+2y+3z=2$ and $x-y+z=3$ and at a distance $2/\sqrt{3}$ from the point $(3,1,-1)$ is:-

A) $5x-11y+z=17$ B) $\sqrt{2}x+y=3\sqrt{2}-1$

C) $x+y+z=\sqrt{3}$ D) $x-\sqrt{y}=1-\sqrt{2}$

Q.32 If the distance of the point $P(1,-2,1)$ from the plane $x+2y-2z=\alpha$, where $\alpha>0$, is 5, the foot of the perpendicular from P to the plane is:-

A) $\left(\frac{8}{3},\frac{4}{3},-\frac{7}{3}\right)$ B) $\left(\frac{4}{3},-\frac{4}{3},\frac{1}{3}\right)$ C) $\left(\frac{1}{3},\frac{2}{3},\frac{10}{3}\right)$ D) $\left(\frac{2}{3},-\frac{1}{3},\frac{5}{2}\right)$

(53)

By:- Niraj Keshri
B.Tech. IIT Kharagpur

Q.33 The point P is the intersection of the straight line joining the points Q(2,3,5) and R(1,-1,4) with the plane $5x-4y-z=1$. If S is the foot of the perpendicular drawn from the point T(2,1,4) to QR, the length of the line segment PS is:-

A) $1/\sqrt{2}$ B) $\sqrt{2}$ C) 2 D) $2\sqrt{2}$

Q.34 Let P(3,2,6) be a point in space and Q be a point on the line $\vec{r} = (\hat{i}-\hat{j}+2\hat{k})+\lambda(-3\hat{i}+\hat{j}+5\hat{k})$

Then the value of λ for which the vector $\vec{PQ}$ is parallel to the plane $x-4y+3z=1$ is:-

A) $1/4$ B) $-1/4$ C) $1/8$ D) $-1/8$

Q.35 The equation of the plane containing the straight line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is:-

A) $x+2y-2z=0$ B) $3x+2y-2z=0$

C) $x-2y+z=0$ D) $5x+2y-4z=0$

Q.36 The perpendiculars are drawn from points on the line $\frac{x+2}{2} = \frac{y+1}{-1} = \frac{z}{3}$ to the plane $x+y+z=3$. The feet of the perpendiculars lie on the line:

A) $\frac{x}{5} = \frac{y-1}{8} = \frac{z-2}{-13}$ B) $\frac{x}{2} = \frac{y-1}{3} = \frac{z-2}{-5}$

C) $\frac{x}{4} = \frac{y-1}{3} = \frac{z-2}{-7}$ D) $\frac{x}{2} = \frac{y-1}{-7} = \frac{z-2}{5}$

Q.37 Two lines $L_1: x=5, \frac{y}{3-\alpha} = \frac{z}{2}$ and $L_2: x=\alpha, \frac{y}{-1} = \frac{z}{2-\alpha}$ are coplanar, then α can take the values

A) 1 B) 2 C) 3 D) 4

Q.38 Let P be the image of the point $(3, 1, 7)$ w.r.t the plane $x-y+z=3$. Then the equation of the plane passing through P and containing the straight line $\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$ is:-

A) $x+y-3z=0$ B) $3x+z=0$

C) $x-4y+7z=0$ D) $2x-y=0$

——X——X——